Sciencia Acta Xaveriana
An International Science Journal
ISSN. 0976-1152

Volume 4
No. 2
pp. 59-64
September 2013

Covering Cover Pebbling Number for Jahangir Graph $J_{2,m}$

A. Lourdusamy,
Department of Mathematics,
St. Xavier 's College (Autonomous),
Palayamkottai - 627 002, India.
E-mail: lourdusamy15@gmail.com

Abstract : Let G be a connected graph. Let p be the number of pebbles distributed on the vertices of G. A pebbling move is defined by removing two pebbles from one vertex and put a pebble on an adjacent vertex. The covering cover pebbling number, $\sigma(G)$, is the least p such that after a sequence of pebbling moves, the set of vertices should form a covering for G from every configuration of p pebbles on the vertices of G. In this paper, we determine the covering cover pebbling number for Jahangir graph $J_{2,m}$.

Key words : Graph pebbling, cover pebbling, covering, square of a cycle.

2009 Mathematics Subject Classification: 05C99.

(Received July 2013, Accepted August 2013)

1 Introduction

Pebbling, one of the latest evolutions in graph theory proposed by Lakarias and Saks, has been the topic of vast investigation with significant observations. Having Chung [1] as the forerunner to familiarize pebbling into writings, many other authors too have developed this topic. Hulbert published a survey of graph pebbling [3]. Given a connected graph G, distribute certain number of pebbles on its vertices in some configuration. Precisely, a configuration on a graph G, is a function from V(G) to $N \cup \{0\}$ representing a placement of pebbles on G. The *size* of the configuration is the total number of pebbles placed on the vertices. *Support vertices* of a configuration C are those on which there is at least one pebble on C. In any configuration, if all the pebbles are placed on a single vertex, it is called a *simple configuration*. A pebbling move is the removal of two pebbles from one vertex and the addition of one pebble to an adjacent vertex. In (regular) pebbling, the target is selected and the aim is to move a pebble to the target vertex. The minimum number of pebbles, such that regardless of their initial placement and regardless of the target vertex, we can pebble that target vertex is called the pebbling number of G, denoted by f(G). In cover pebbling, the aim is to cover all the vertices with pebbles, that is, to move a pebble to every vertex of the graph simultaneously. The minimum number of pebbles required such that regardless of their initial placement on G, there is a sequence of pebbling moves, at the end of which, every vertex has at least one pebble on it, is called the cover pebbling number of G. In [2], Crull et al. determine the cover pebbling number for complete graphs, paths, and trees. Hulbert and Munyan [4], determine the cover pebbling number of the d-cube. A set $K \subseteq V(G)$ is a covering if every edge of G has at least one end in K. The covering cover pebbling number of G, denoted by $\sigma(G)$, is the smallest number of pebbles, such that, however the pebbles are initially placed on the vertices of the graph, after a sequence of pebbling moves, the set of vertices with pebbles forms a covering of G. In [6], Lourdusamy et al., have introduced this concept covering cover pebbling number and have determined the covering cover pebbling number for complete graphs, paths, wheel graphs, complete r-partite graphs and binary trees. Lourdusamy et al., have also determined more results on covering cover pebbling number in [5, 7, 8, 9] .

In the next section, we determine the covering cover pebbling number of Jahangir graph $J_{2,m}$.

2. Covering cover pebbling number of Jahangir graph $J_{2,m}$.

Definition 2.1: Jahangir graphs $J_{n,m}$ for $m \geq 3$ is a graph on nm+1 vertices, that is, a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other of C_{nm}.

We now give the labeling for $J_{2,m}$. Let v_{2m+1} be the label of the center vertex and v_1, $v_2, \ldots, v_{2m}$ be the label of the vertices that incident clockwise on cycle C_{2m} so that deg $(v_1) = 3$. Figure 1 shows the Jahangir graph $J_{2,3}$ with its labeling.

Consider the sets $S_1 = \{v_1, v_3, \ldots, v_{2m-1}\}$ and $S_2 = \{v_2, v_4, v_6, \ldots, v_{2m}\}$. Clearly the sets S_1 and $S_2 \cup \{v_{2m+1}\}$ are the covering sets for $J_{2,m}$ with every edge incident with exactly one vertex.

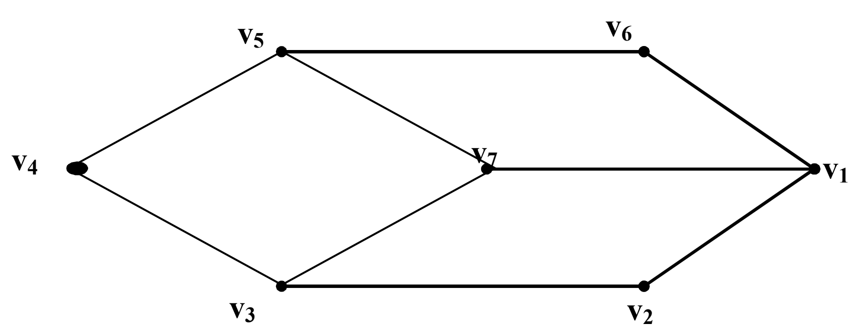

Figure 1: The Jahangir graph $J_{2,3}$

Lemma 2.2: The value of $\sigma(J_{2,m})$ is attained when the original configuration consists of placing all the pebbles on a single vertex of S_2.

Proof :
Claim: No worst configuration consists of pebbled vertices in $S_1 \cup \{v_{2m+1}\}$.
Consider a worst configuration C.

Case (i): Suppose, if v_{2m+1} is pebbled and the vertices of S_1 are unpebbled in C.

Consider the effect of relocating all the pebbles at v_{2m+1} to a single vertex of S_1. One would now require more pebbles to cover the edges of $J_{2,m}$ – a contradiction.

Case (ii): More than one vertices of S_1 and the v_{2m+1} are pebbled in C.

Now, we relocate all the pebbles which are on the vertices of S_1 and v_{2m+1} to a single vertex of S_1. One would now require more pebbles to cover the edges of $J_{2,m}$ – a contradiction.

Case (iii): More than one pebbled vertices in S_1 and v_{2m+1} is unpebbled in C.

Clearly, we need more pebbles to cover all the edges, if we relocate all the pebbles on the vertices of S_1, to a single vertex of S_1 – a contradiction.

From these three cases, we conclude that, no worst configuration consists of more than one pebbled vertex in S_1 and also v_{2m+1} can't be pebbled.

Thus, every worst configuration consists of pebbled vertices only in $S_2 \cup \{v_i\}$, where $v_i \in S_1$ is the only pebbled vertex in S_1.

However, we get a contradiction.

Since if we relocate the pebbles at $v_i \in S_1$ to any one of the adjacent vertices of v_i, we need more pebbles to cover the edges of $J_{2,m}$.

$\therefore$ No worst configuration consists of pebbled vertices in $S_1 \cup \{v_{2m+1}\}$. Hence the claim.

So, any worst configuration consists of pebbled vertices only in S_2 (from the claim). Next assume that a worst configuration consists of more than one pebbled vertices in S_2. Once again, we get a contradiction, since we need more pebbles to cover the edges of $J_{2,m}$ after relocating all the pebbles to a single vertex of S_2. The statement follows. ■

Since placing all the pebbles on a single vertex of S_2 is a worst case, we now determine the value of $\sigma(J_{2,m})$.

Theorem 2.3: Let $J_{2,m}$ be a Jahangir graph on 2m+1 vertices. Then, $\sigma(J_{2,m}) = 4(2m - 3)$

Proof: Consider the graph $J_{2,m}$ ($m \geq 3$). Without loss of generality, Let us assume that v_2 is our target vertex. If we put one pebble each either at the vertices of S_1 or $S_2 \cup \{v_{2m+1}\}$ from v_2, then clearly we covered all the edges of $J_{2,m}$. Since S_1 and $S_2 \cup \{v_{2m+1}\}$ are covering sets for $J_{2,m}$.

But, if we pebble the each vertex of $S_2 \cup \{v_{2m+1}\}$, we need more pebbles to put a pebble at each vertex of $S_2 \cup \{v_{2m+1}\}$ from v_2. So, we take the set S_1 and put a pebble each at the vertices of S_1 from v_2, where $S_1 = \{v_1, v_3, v_5, \ldots, v_{2m-1}\}$ and $|S_1| = m$.

Since, $d(v_1, v_2) = d(v_3, v_2) = 1$ and $d(v_2, v_i) = 3$ for all $i = 5, 7, \ldots, 2m - 1$, we need $2^1 + 2^1 + (m - 2)\, 2^3$ pebbles at v_2 to cover the edges of $J_{2,m}$. That is, we need $4 + 8(m - 2) = 4 + 8m - 16 = 8m - 12 = 4(2m - 3)$ pebbles at v_2.

$\therefore\ \sigma(J_{2,m}) = 4(2m - 3)$. ■

References

[1] F.R.K. Chung, Pebbling in hypercubes, SIAM J Discrete Math 2 (1989), 467-472.

[2] Crull, Cundiff, Feltman, Hulbert, Pudwell, Szaniszlo, Tuza, The cover pebbling number of graphs, Discrete Math. 296 (2005), 15-23.

[3] G. Hulbert, A survey of graph pebbling, Congr. Numer. 139(1999), 41-64.

[4] G. Hulbert and B. Munyan, The cover pebbling number of hypercubes, Bull. Inst.combin.Appl., 47(2006),71-76.

[5] A. Lourdusamy and T. Mathivanan, Covering cover pebbling number for Square of a cycle, Journal of Prime Research in Mathematics, 8 (2012), 102-105.

[6] A. Lourdusamy and A. Punitha Tharani, Covering cover pebbling number, Utilitas Mathematica (2009), 41-54.

[7] A. Lourdusamy, S. Samuel Jayaseelan and T. Mathivanan, Covering cover pebbling number for Even cycle lollipop, Gen. Math. Notes, 5(2) (2011), 24-41.

[8] A. Lourdusamy, S. Samuel Jayaseelan and T. Mathivanan, Covering cover pebbling number for Odd cycle lollipop, Int. J. Pure Appl. Sci. Technol., 5(2) (2011), 77-83.

[9] A. Lourdusamy, S. Samuel Jayaseelan and T. Mathivanan, Covering cover pebbling number for Sun, Int. J. Pure Appl. Sci. Technol., 5(2) (2011), 109-118.

[10] L. Pachter, H. S. Snevily and B. Voxman, On Pebbling Graphs, Congr. Numer. 107 (1995), 65-80.